Chapter 1: Stellar Magnitudes, Colors and Spectra

1-1 Apparent Magnitudes

1-1-1 Apparent Magnitudes

- The simplest quantity for a star is its *brightness*, which can be measured by the power flux received on Earth

- In modern astronomy, the lights are measured by the light sensitive devices such as Charge-Coupled Device (CCD), photomultipliers, etc., which are calibrated by standard light source with energy flux is known.

- It was not easily done by the ancient astronomers. However similar process had been done for a long time by comparing the brightness of stars.

- In principle, we have to pick up a star as standard (called *standard star,* similar to doing the calibration) and compare the brightness with the target.

- No variable star can be chosen as the standard star.

- A number of stars have been selected as standard stars (e.g. Landholt Photometric Standard Star Catalog).

- The ancient astronomers used naked eyes to observe the night sky and called the brightest stars first magnitude, the second brightest stars second magnitude etc.

- Interestingly, the response of the human eyes to the dim light is *logarithmic*.

- Thus, the apparent magnitude is proportional to the logarithm of the energy flux.

1-1-2 Apparent Magnitude and Energy Flux

- **The apparent magnitudes refer to the energy received above the atmosphere.**

- All the stars are very far away from the Earth. The lights from them can be considered as parallel light.

- The total energy receive from the light with specified wavelength band and perpendicular to the light beam of a star proportional to:

    - Exposure (integration) time
    - Area

- Thus, the (energy) flux has a unit of energy per unit area per unit time (e.g. erg cm^{-2} s^{-1})

- Note:

- In the photometry, the λ here usually refers to *wavelength band* (e.g. B, V. I, g, r) instead of specified wavelength since for a specified wavelength, the flux is approach to zero.
- In the spectroscopy, astronomers prefer to use *flux density* whose unit is energy per unit area per unit time per unit wavelength (or frequency) to express the flux of a specified wavelength (or frequency)
- The astronomers studying by the different wavelength band would use different unit to express the flux density, for example, radio astronomers: jansky (Jy) or millijansky (mJy), 1Jy=10^{-26} watts per square meter per hertz; X-ray/ γ -ray astronomers: erg cm^{-2} s^{-1} keV^{-1}.

- The relation between apparent magnitude and energy flux

$$m_\lambda = -2.5 \cdot \log f_\lambda + const$$
$$f_\lambda = const' \times 10^{-0.4 m_\lambda}$$
$$\Rightarrow \text{ Magnitude difference}$$
$$m_\lambda(1) - m_\lambda(2) = -2.5 \log \frac{f_\lambda(1)}{f_\lambda(2)}$$

- It does not determine the magnitude of the stars unless one of them is given.

- **The star Vega always has the magnitude zero by definition; no matter what the wavelength band it is** (the only one exception is bolometric magnitude).

1-2 Stellar Colors

- A star emits light with all wavelengths. The question which one (longer or shorter) emits more than the other one.
- In principle, to see the wavelength distribution of the emission from a star (i.e. spectrum), a spectrogram is required.
- However, it is more difficult to get the spectrum of a star. It requires a larger telescope.
- It would be easier to compare the magnitudes of different wavelength bands.
- The stellar color index is defined by the magnitude difference of the two wavelength bands (usually neighboring bands).
- For example, $m_B - m_V = B - V$.
- By definition, the color index of Vega is 0.
- A star redder than Vega has positive color index.
- A star bluer than the Vega has negative color index.
- The Sun's color index: B-V= 0.63, which is redder than Vega.
- **The blackbody is a good approximation to describe the radiation of a star. The color is also an index for stellar temperature.**

  Blackbody Radiation : Wein's displacement law

  $\lambda_{\max} T = \text{constat} = 0.28973 \text{ cm deg}$

  where $\lambda_{\max}$ is the wavelength with the maximum flux

- Higher temperature ➔ shorter $\lambda_{\max}$ ➔ bluer.
- Lower temperature ➔ longer $\lambda_{\max}$ ➔ redder
- Large color index ➔ redder ➔ cooler
- Smaller color index ➔ bluer ➔ hotter.
- Note that the blackbody is only an approximation. The spectrum for a real star still deviates from a blackbody. It can be easily seen by the color/color diagram of the real stars and blackbody. One of the purposes of this course is to understand why the stars' energy distributions are different from a blackbody.

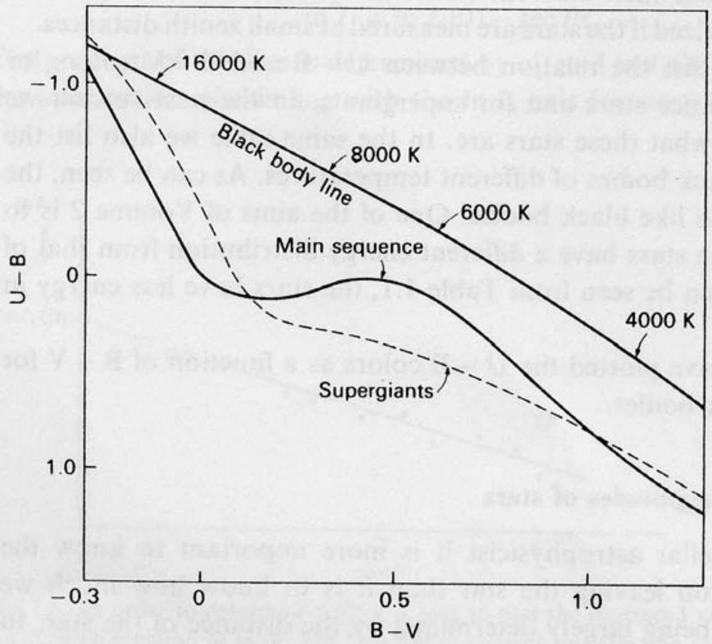
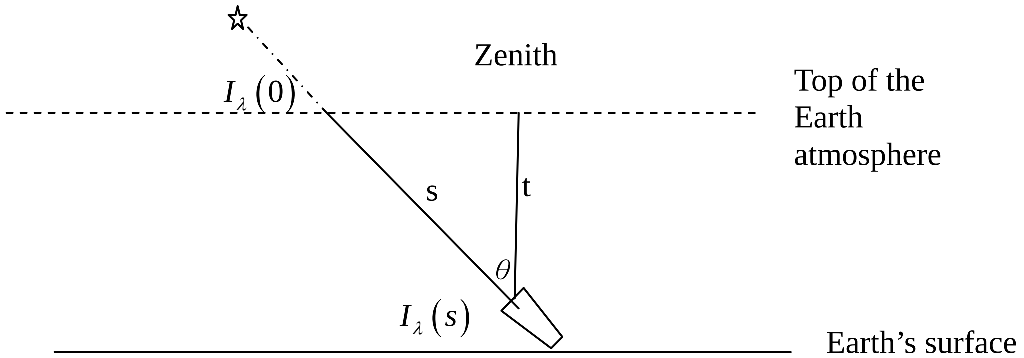


Fig. 1.7. The two-color diagram for the U − B, B − V colors for stars and for black bodies.

1-3 Correction for the Absorption in the Earth's Atmosphere

- As discussed in section 1-2, the apparent magnitude is refer to the energy received above the atmosphere.

- Thus the correction for the absorption in the Earth atmosphere is required.



$$
\begin{aligned}
&I_\lambda(s) = I_\lambda(0)\cdot e^{-\tau_\lambda(s)} \\
&\tau_\lambda(s) = \int_0^s \rho\kappa_\lambda \cdot ds' \\
&s = \sec\theta \cdot t \\
&\Rightarrow ds' = \sec\theta \cdot dt' \\
&\tau_\lambda(s) = \int_0^s \rho\kappa_\lambda \cdot ds' = \sec\theta \int_0^t \rho\kappa_\lambda \cdot dt' = \sec\theta \cdot \tau_\lambda(t) \\
&I_\lambda(t,\theta) = I_\lambda(0)\cdot e^{-\sec\theta\cdot\tau_\lambda(t)}
\end{aligned}
$$

- The intensity can be measured by the calibrated optical device (e.g. CCD).

- To get the optical depth $\tau_\lambda(t)$, we only have to, in principle, measure the intensities of the star in two different positions

$$I_\lambda(t,\theta) = I_\lambda(0)\cdot e^{-\sec\theta\cdot\tau_\lambda(t)}$$

$$\ln I_\lambda(t,\theta) = \ln I_\lambda(0) - \sec\theta\cdot\tau_\lambda(t)$$

$$\begin{cases}\ln I_\lambda(t,\theta_1) = \ln I_\lambda(0) - \sec\theta_1\cdot\tau_\lambda(t)\\ \ln I_\lambda(t,\theta_2) = \ln I_\lambda(0) - \sec\theta_2\cdot\tau_\lambda(t)\end{cases}$$

$$\Rightarrow \ln I_\lambda(t,\theta_1) - \ln I_\lambda(t,\theta_2) = (\sec\theta_2 - \sec\theta_1)\cdot\tau_\lambda(t)$$

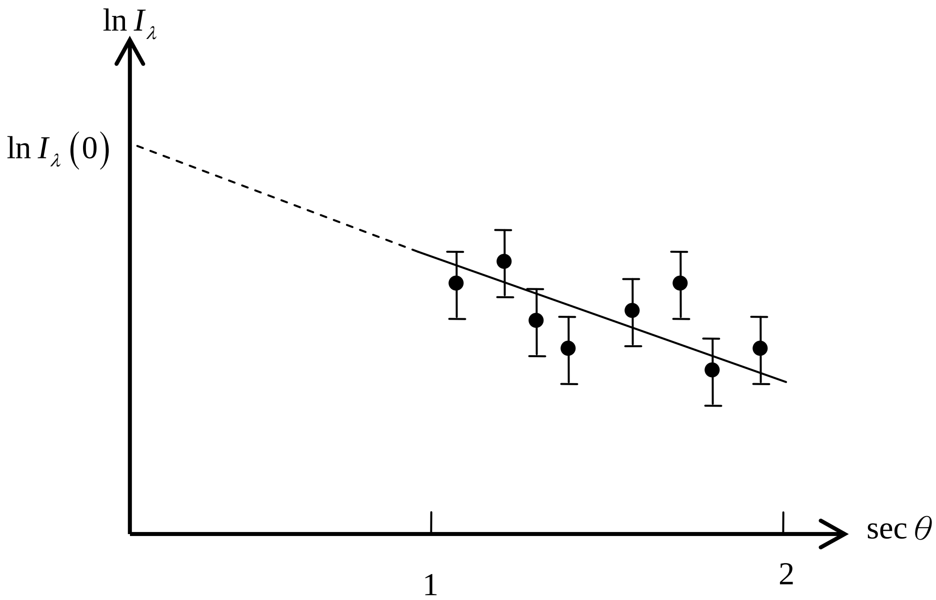
$$\tau_\lambda(t) = \frac{\ln I_\lambda(t,\theta_1) - \ln I_\lambda(t,\theta_2)}{\sec\theta_2 - \sec\theta_1}$$

$$I_\lambda(s,\theta) = I_\lambda(0)\cdot e^{-\sec\theta\cdot\tau_\lambda(t)}$$

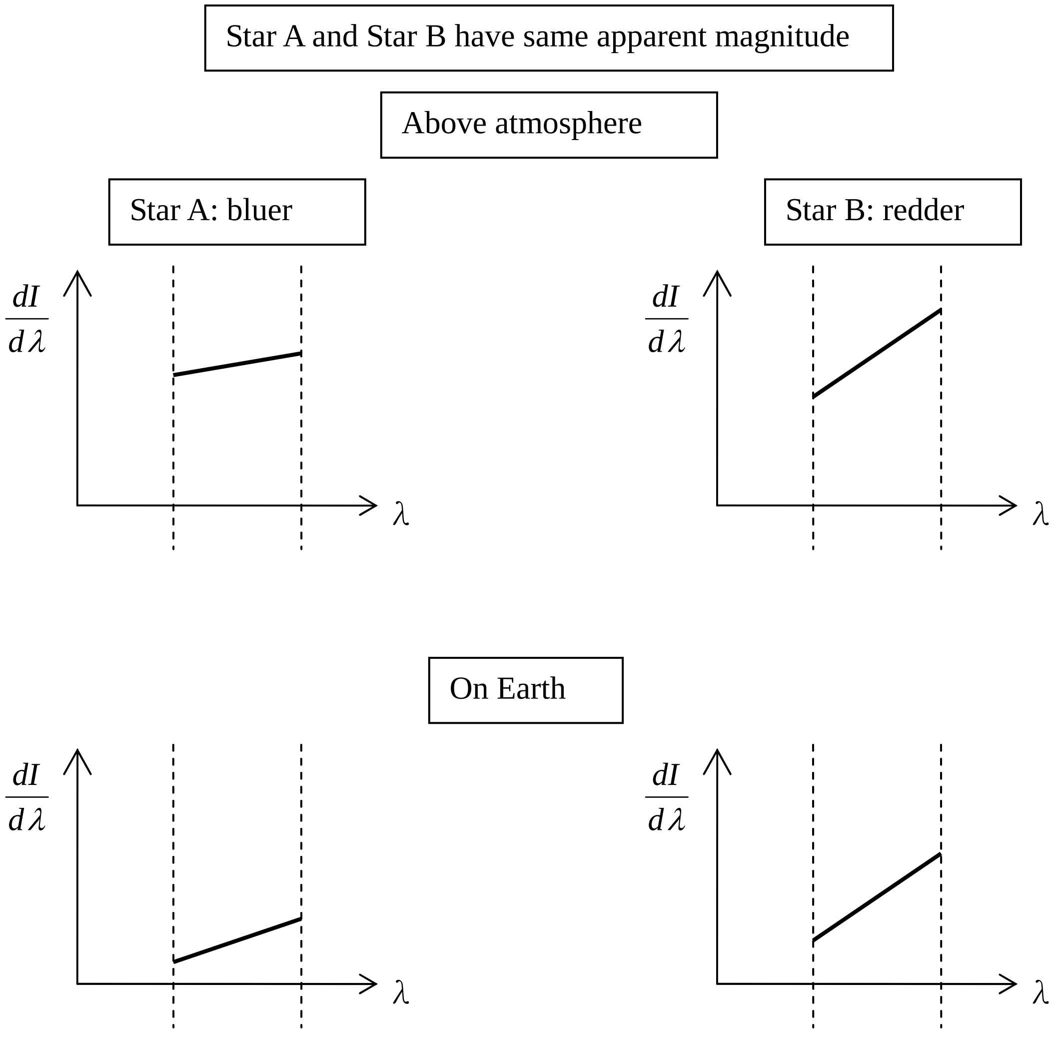
$$-2.5\log I_\lambda(t,\theta) = -2.5\log I_\lambda(0)\cdot -2.5\log e^{-\sec\theta\cdot\tau_\lambda(t)}$$

$$\Rightarrow m_\lambda(t,\theta) = m_\lambda(0) + 2.5\sec\theta\cdot\tau_\lambda(t)\log e$$

- However, no measurement with no error. We thus measure many points and then fit a straight line. The slope is $\tau_\lambda(t)$ and the intercept is $\ln I_\lambda(0)$.



- In fact, there are some problems with the derivation above:
  - For large angle, the curvature of atmosphere has to be considered. Furthermore, the light beam is bent due to the refraction. Thus $\tau_\lambda(t,\theta)/\tau_\lambda(t,0) \neq \sec\theta$.
  - The actual ratio called air mass ($M(\theta) = \tau_\lambda(t,\theta)/\tau_\lambda(t,0)$) but if $\theta < 60^\circ$ $(\sec\theta < 2)$, the difference is small and can be neglected.
  - The derivation above is for a specified wavelength. For the broad band (UBVR), $\tau_\lambda(t)$ would be wrong unless κ_λ is constant over the band.
  - For example, if $\tau_\lambda(t)$ is obtained by a bluer star, the value would be too large for a redder star.



$$I_\lambda(A) < I_\lambda(B)$$
$$\Rightarrow \tau_\lambda(A) > \tau_\lambda(B)$$

- However, please recall that we get the apparent magnitude by comparing with the standard star. If there is a standard star in the field of view (which is usually small), $\theta(\text{standard}) \approx \theta(\text{target}) = \theta$

$$m_\lambda(t,\theta;\text{target}) = m_\lambda(0;\text{target}) + \sec\theta \cdot \tau'_\lambda(t)\log e$$
$$m_\lambda(t,\theta;\text{standard}) = m_\lambda(0;\text{standard}) + \sec\theta \cdot \tau'_\lambda(t)\log e$$
$$m_\lambda(t,\theta;\text{target}) - m_\lambda(t,\theta;\text{standard}) = m_\lambda(0;\text{target}) - m_\lambda(0;\text{standard})$$

- If the standard star is not in the field of view, extinction correction is required.
- What if the colors of target and the standard star are different even if they are in the same field of view?

1-4 Luminosities of Stars and Absolute Magnitudes

1-4-1 Luminosity

- Luminosity: the total amount of radiative energy leaving the stellar surface per unit time is called luminosity.

- Suppose the power per unit area emitted from the star surface is F .

$$L = 4\pi R^2 F$$
$$R = \text{radius of the star}$$

- For an observer at distance d away from the star, from the conservation of energy (e.g. no absorption by the interstellar matter)

$$L = 4\pi d^2 f$$
$$\Rightarrow f = F\frac{R^2}{d^2} = \frac{L}{4\pi d^2}$$

- The luminosity above refers to the total energy radiating from a star. However, the inverse square law is also true for the radiation of specified wavelength or wavelength band.

$$f_\lambda = \frac{L_\lambda}{4\pi d^2}$$

1-4-2 Absolute Magnitude

- From discussion in 1-4-1, the apparent magnitude is highly depend on the distance

$$\begin{aligned} m_\lambda &= -2.5\log f_\lambda + const \\ &= -2.5\log\frac{L_\lambda}{4\pi d^2} + const \\ &= -2.5\log L_\lambda + 5\log d + 2.5\log 4\pi + const \\ &= -2.5\log L_\lambda + 5\log d + const' \end{aligned}$$

- However, in the optical astronomy, we seldom use luminosity to express the brightness of a star but use the absolute magnitude

- Absolute magnitude: the apparent magnitude if the star were located at the distance of 10 pc from the Earth

$$
\begin{aligned}
&M_\lambda = -2.5\log L_\lambda + 5\log d_{10} + const' \\
&d_{10} = 10 \text{ pc} \\
&M_\lambda - m_\lambda = 5\log d_{10} - 5\log d \\
&\text{If the distance is in the unit of pc} \\
&M_\lambda - m_\lambda = 5 - 5\log d \\
&\log d = \frac{5 - M_\lambda + m_\lambda}{5} \\
&d = 10^{\frac{m_\lambda - M_\lambda + 5}{5}}
\end{aligned}
$$

- $M_\lambda - m_\lambda$ is called *distance modulus*, which is independent of the wavelength band

- *The color index determined by the absolute magnitude is same as the one derived from the apparent magnitude.*

1-4-3 Bolometric Magnitudes

- Due to the limitation of the detector, as well as the absorption of interstellar media, no telescope can observe all wavelengths radiated from a star.

- The magnitude of all the wavebands (from radio to γ -ray, in principle) is named *bolometric magnitude*, including absolute (denote as M_{bol}) and apparent (denote as m_{bol}).

- We are not able to directly measure the bolometric magnitude but rely on the theoretical extrapolation to estimate the amount of radiation emitted form a star.

- Astronomers usually compare the bolometric magnitude with the magnitude of virtual band (V) using bolometric correction

$$
\begin{aligned}
&M_{bol} = M_V - BC \\
&BC = \text{bolometric correction}
\end{aligned}
$$

- The bolometric correction, by the definition above, is almost always (except some supergiants) positive. However, some astronomer would like to use plus sign for the equation above so the bolometric correction is negative.

- The very blue star (small B-V value) whose radiation dominates in the ultraviolet band has large bolometric correction value. Similarly, the very red star (large B-V value) whose radiation dominates in the infrared has large bolometric correction value.

- The star has minimum BC value when the color index B-V~0.3.

1-4-4 Hertzprung-Russel (H-R) Diagram

- Also called color-magnitude or color-luminosity diagram.

- Color, temperature or other comparable quantity may represent the spectral type as the parameter plotted along the horizontal axis.

- Traditionally the horizontal axis is color (B-V) since it is much easier to obtain it directly through observations. However, using the concept of black body, the larger color index implies low temperature. Therefore, *temperature conventionally decreases towards the right.*

- The absolute magnitude (not the apparent magnitude unless the stars are grouped (e.g. star cluster) so their distances from Earth can be considered identical) or the luminosity are frequently used as the vertical axis.

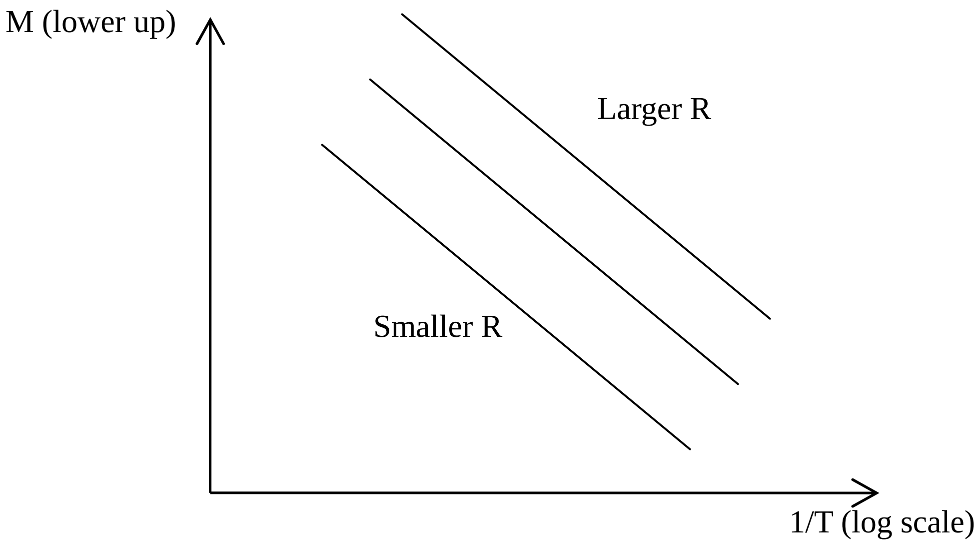
- Note that magnitude conventionally *decreases toward the up*.

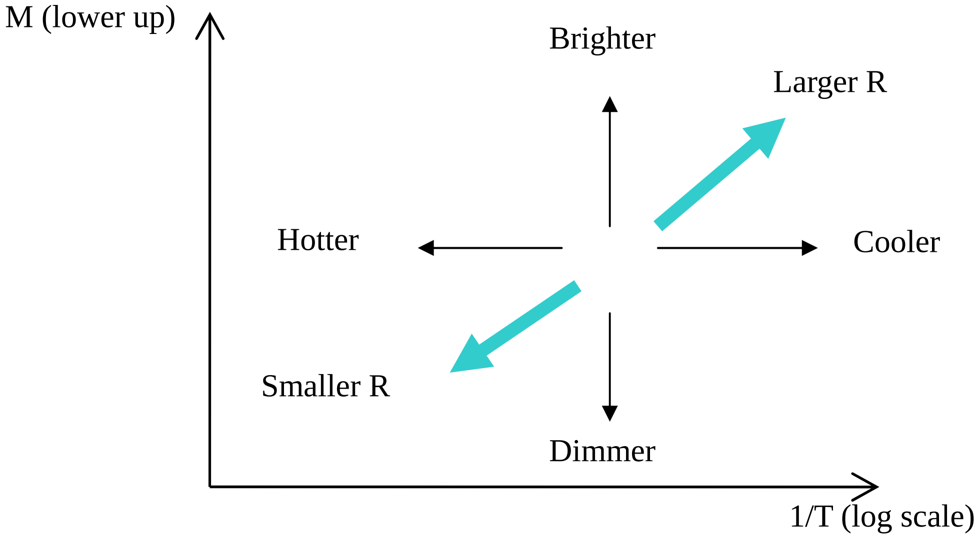
- The HR diagram (relation between temperature and luminosity) gives us clues for the stellar atmospheres, structure and evolution etc.

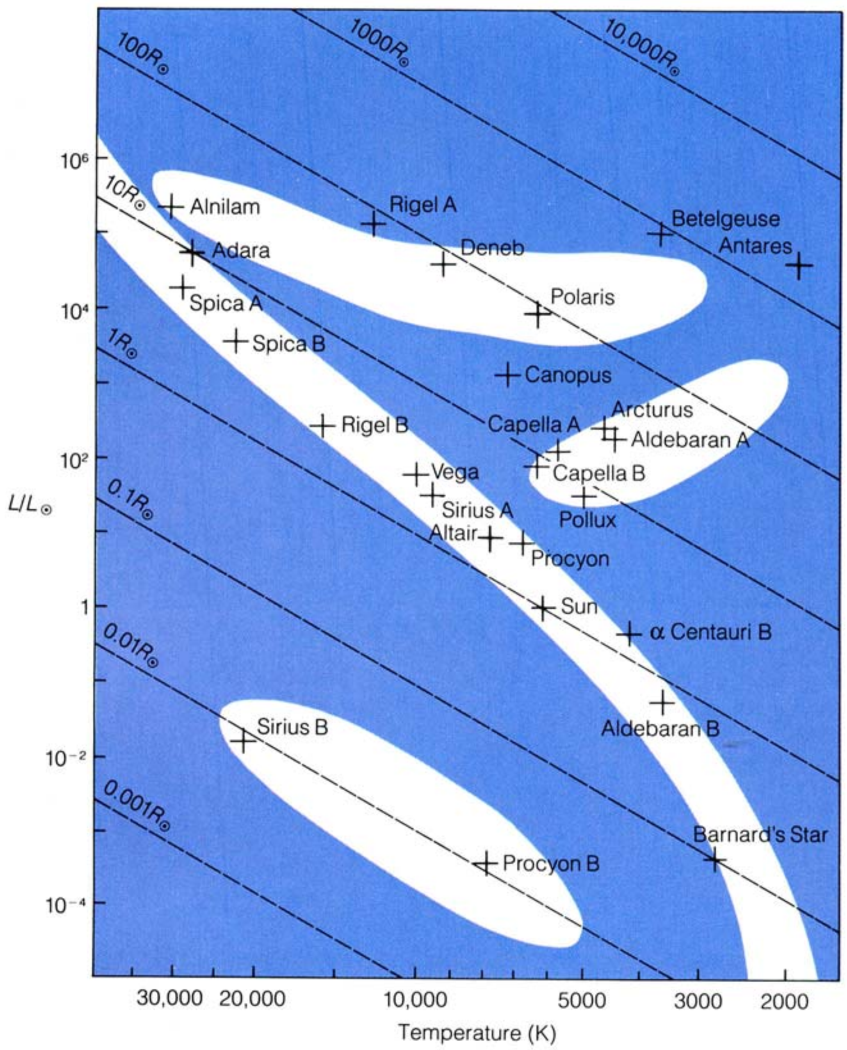
- For example, if the stars exactly obeyed the blackbody radiation:

$$
\begin{aligned}
& L \propto R^2 T^4 \\
& \Rightarrow \log L = 2\log R + 4\log T + const \\
& \Rightarrow \log L = 2\log R - 4\log\left(\frac{1}{T}\right) + const \\
& \Rightarrow -2.5\log L = -5\log R + 10\log\left(\frac{1}{T}\right) + const
\end{aligned}
$$

  - If all the stars had same radius, they would lie on a straight line on the plot of 1/T (log scale) and absolute magnitude (lower up) (i.e. HR diagram)





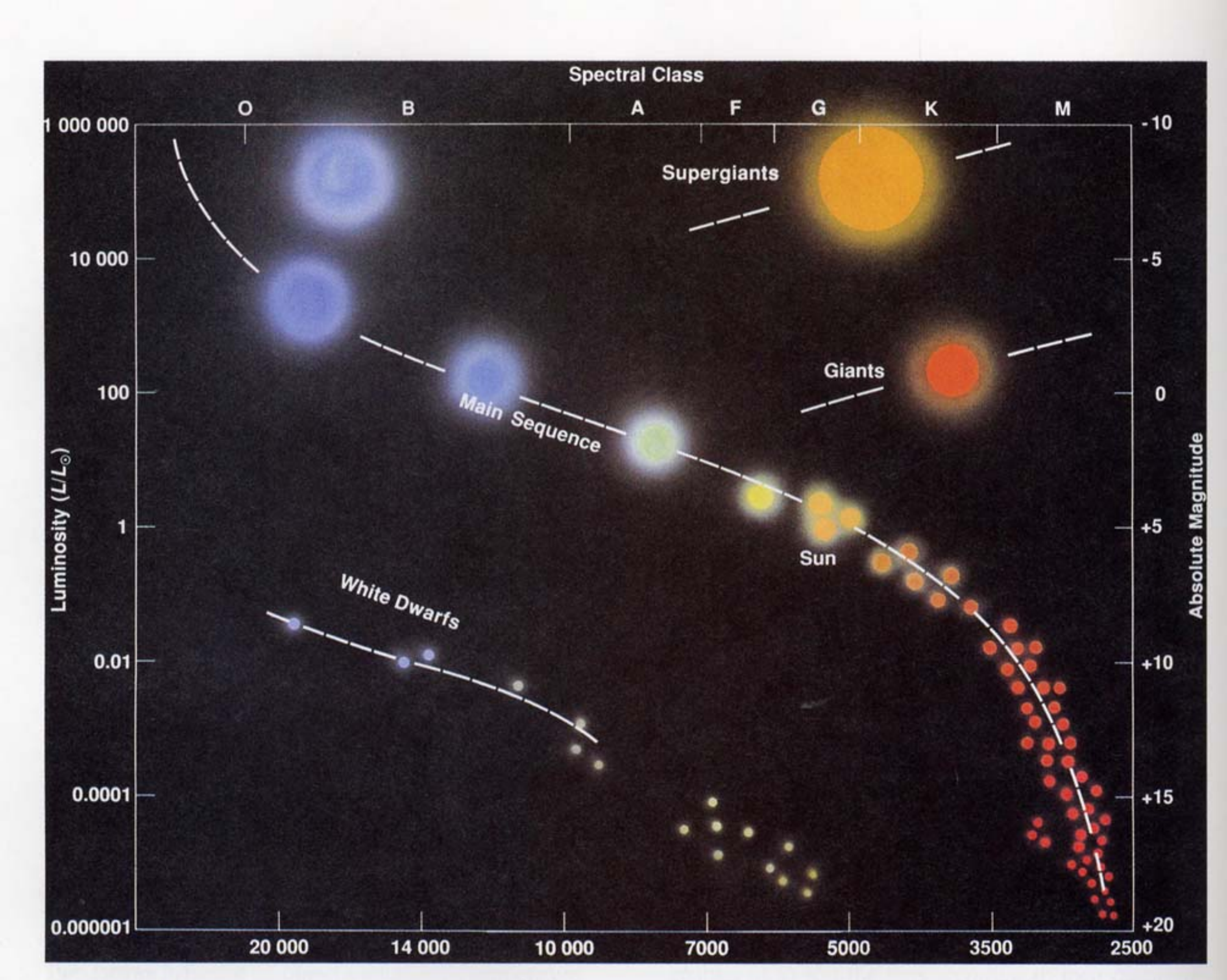


- Interestingly, the stars are not randomly distributed over the HR diagram. A large number of stars lie on a band diagonally from upper left to lower right, the so called *main-sequence stars*. It implies that there must be something to link these stars (what is it?). The stars form a sequence indicates that there is a factor determining the spectral type and luminosity. The factor is mass of the star so it is an effect of mass sequence.

- There are some stars scattered above the main-sequence called giant stars (including subgiants, giants, bright giants and supergiants). With the concept of blackbody, their locations in HR diagram indicate that they are, in general, larger than the main-sequence (that is the why they are named as "giant").

- A small number of stars lies on the lower-left are white dwarfs

  - White – which means hot so they locate at the left side of HR diagram
  - Dwarf – which means dim (or small) so they locate at the lower side of the HR diagram.
  - Using the concept of black body, the white dwarf must have small radius (about the size of the Earth).



1-5 Stellar Spectra

- Most of the stars' (visible) spectra show strong absorption lines.

- Now we have understood that they are owing to the absorption of hydrogen atoms in the stellar atmosphere.

  - The energy level of hydrogen atom

$$E_n = -\frac{13.6\text{eV}}{n^2}$$

$$h\nu = \frac{hc}{\lambda} = \Delta E(n \to m) = E_m - E_n = 13.6\text{eV}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$\lambda = \frac{hc}{13.6\text{eV}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)} = \frac{912\ \mathring{A}}{\frac{1}{n^2} - \frac{1}{m^2}}$$

	Lyman n=1	Balmer n=2	Paschan n=3
m=2	1216 Å	******	******
m=3	1026 Å	6562.8 Å (H α)	******
m=4	972 Å	4864 Å (H β)	18176 Å
m=5	950 Å	4342.9 Å (H γ)	12825 Å
m=6	938 Å	4104 Å (H δ)	10944 Å
m= ∞	912 Å	3648 Å	8208 Å

  - Visible light: $7500\,\mathring{A} > \lambda > 4000\,\mathring{A}$
  - Lyman lines are basically ultraviolet.
  - Paschan lines are basically infrared.
  - Balmer lines mostly lie on visible band.

- The spectral lines (visible) we see in the spectra of stars are Balmer lines.

- The astronomers used to use the strength of these lines to classify the spectra type of the stars
  - The one with the strongest Balmer absorption lines called A stars
  - The one with the second strongest Balmer absorption lines called B stars
  - Etc.

- However, if we rearrange these classes of stars by their color (B-V), the O stars are the bluest and then B, A ….
  - Spectral type ordered in sequence of decreasing effective temperature (increasing the

color index): OBAFGKM(RNS)

    - Oh, Be A Find Girl (Guy), Kiss Me ! (Right Now! Smack)
  - Each of the main class is further broken down into up 10 subdivision, indicated by the number of 0 to 9 (e.g. A0, G2, K5)

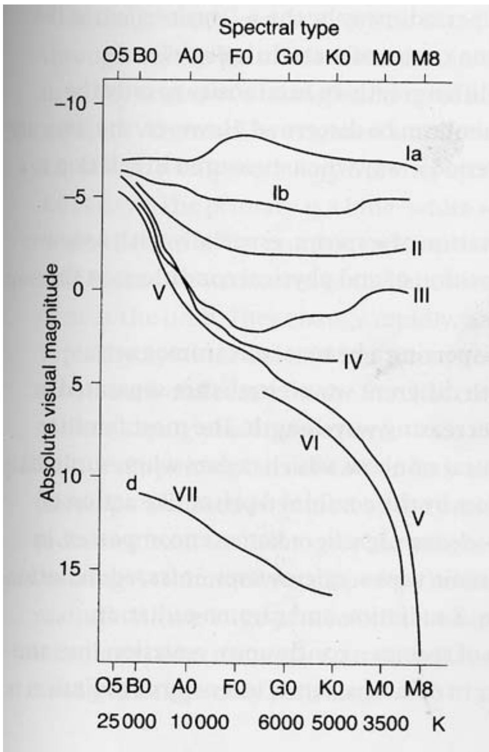
- Principle features of the main spectral types

Spectral type	Temperature	Principle feature of visible spectrum
O	>25000K	Relatively few absorption lines. Lines of ionized He, doubly ionized N, triply ionized Si. H lines weak
B	11000-25000K	Lines of neutral He, singly ionized O and Mg. H lines stronger than O stars.
A	7500-11000	Strongest H lines. Lines of singly ionized Mg, Si, Fe, Ti, Ca, etc. and some of neutral metals
F	6000-7500K	H lines weaker and neutral metal lines stronger than in A stars. Lines of singly ionized Ca, Fe, Cr.
G	5000-6000K	Lines of ionized Ca most conspicuous feature. Many lines of ionized and neutral metals.
K	3500-5000K	Neutral metal lines predominate
M	<3500K	Strong lines of neutral metals and molecular bands of TiO

- Note that astronomers usually call the elements with atomic number greater than 5 the “metals”, even if some of them are not metals, like N, O.

- Wrong names but still used today
  - Early type star: blue/hot star
  - Late type star: red/cool star
  - These names have nothing to do with the age.

- Luminosity class: indicated by capital Roman numerals.
  - Ia: Luminous supergiants
  - Ib: Less luminous supergiants
  - II: Bright giants
  - III: Normal giants
  - IV: Subgiants
  - V: Dwarf/Main-sequence
  - VI: Subdwarf (rarely used)
  - VII White dwarf (rarely used)



- Various prefixes and suffixes are used to give additional information about the spectra, for example
  - c sharp line
  - d dwarf/main-sequence stars
  - D white dwarf
  - e emission
  - em emission in metal lines
  - ep peculiar emission
  - eq emission with shorter wavelength absorption
  - f emission by helium and neon in O stars
  - g giant
  - k interstellar lines
  - m strong metallic lines
  - n diffuse lines
  - nn very diffuse lines
  - p peculiar spectrum
  - s sharp lines
  - sd subdwarf
  - wd white dwarf
  - wk weak lines
  - etc.
- For example, a B3 giant with emission lines: B3IIIe